The Effects of Amplification Bias in

Gravitational Microlensing Experiments[1]

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ABSTRACT

Although a source star is fainter than the detection limit imposed by crowding, it is still possible to detect an event if the star is located in the seeing disk of a bright star is and gravitationally amplified: amplification bias. Using a well-constrained luminosity function, I show that $\sim 40\%$ of events detected toward the Galactic bulge are affected by amplification bias and the optical depth might be overestimated by a factor ~ 1.7. In addition, I show that if one takes amplification bias into consideration, the observed time scale distribution matches significantly better, especially in the short time-scale region, with the distribution expected from a mass-spectrum model in which lenses are composed of the known stellar population plus an additional population of brown dwarfs than it is without the effect of the amplification bias.

Subject headings: Cosmology: gravitational lensing, Stars: luminosity function,

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1. Introduction

Many candidates of Massive Astrophysical Compact Halo Objects (MACHOs) are being detected by the MACHO (Alcock et al. 1993, 1996a), EROS (Aubourg et al. 1993, 1995), OGLE (Udalski et al. 1992, 1994), and DUO (Alard 1996) groups. To maximize the event rate, the searches are being carried out toward very dense star fields, e.g., the Galactic bulge and Magellanic Clouds, in which the detection limit is set by crowding. However, it is possible to detect a lensing event with a source star that is below the detection limit, and thus unresolved, provided that the star is located in the seeing disk of a bright star and is gravitationally amplified. This effect is known as "amplification bias" (Blanford & Narayan 1992; Narayan & Wallington 1994). I will call events of this type and the corresponding source stars below the detection limit "faint events" and "faint stars", respectively. Current lensing experiments are adopting an observational strategy in which they construct templates of resolved stars to allow fast photometric comparison of source star luminosities. It may then appear as if faint events cannot be detected because they are not registered in the template (Bouquet 1993). However, in the very dense field in which nearly all stars are blended, the amplified flux of a faint star will increase the flux of a nearby bright star, and thus an event seemingly with the bright star as a source star can be detected. I will call the stars registered in the template image "bright stars" as opposed to "faint stars". Therefore, the current lensing searches suffer from amplification bias (Nemiroff 1994).

In this paper, using a well-constrained luminosity function (LF) I show that $\sim 40\%$ of events detected toward the Galactic bulge are affected by amplification bias and that the optical depth might be overestimated by a factor ~ 1.7. In addition, I show that by taking amplification bias into consideration, the observed time scale distribution matches significantly better, especially in the short time-scale region, with the distribution expected from a mass spectrum model in which the lenses are composed of the known stellar population plus an additional population of brown dwarfs than it does without taking account of amplification bias.

2. Faint Lensing Events

For the detection of a faint event with a faint star flux $L < L_{DL}$, the amplification should satisfy the condition

$$\frac{F}{F_0} = \frac{L_b + A_{\min}L}{L_b + L} = \frac{3}{\sqrt{5}}, \tag{2.1}$$

resulting in the minimum required amplification of

$$A_{\min} = \frac{(3/\sqrt{5})(L_b + L) - L_b}{L}. \tag{2.2}$$

Here L_b and L_{DL} are the fluxes of the blended bright star and the detection limit. The factor $3/\sqrt{5}$ in above equations is included so that an event can be detected when it is amplified more than $A = 3/\sqrt{5}$ of the combined flux of the bright and faint stars. The amplification is related to the lensing parameters by

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}; \qquad u^2 = \beta^2 + \frac{(t - t_0)^2}{t_e^2}, \tag{2.3}$$

where u is the lens-source separation in units of the Einstein ring radius r_e, β is the impact parameter, t_0 is the time of maximum amplification. The Einstein ring radius is related to the physical parameters of a lens by

$$r_e = \left(\frac{4GM_L}{c^2} \frac{D_{ol}D_{ls}}{D_{os}}\right)^{1/2}, \tag{2.4}$$

where M_L is the mass of the lens, and D_{ol}, D_{ls}, and D_{os} are the distances between the observer, lens, and source. The Einstein time scale is related to r_e by $t_e = r_e/v$, where v is the transverse speed of the lens relative to the observer-source line of sight. The maximum allowed impact parameter for detection is related to the minimum required amplification by

$$\beta_{\max} = \left[2\left(1 - A_{\min}^{-2}\right)^{-1/2} - 2\right]^{1/2}. \tag{2.5}$$

For faint events, one can detect only the portion of a light curve above the threshold L_b and the event therefore mimics that of a shorter time-scale event. Then what is measured is not t_e but the effective time scale t_{eff}, which is the half of the duration of an event above the threshold. The ratio between these two time scales is

$$\eta(\beta) = \frac{t_{\mathrm{eff}}}{t_e} = \left[\beta_{\max}^2 - \beta^2\right]^{1/2} \le 1.0. \tag{2.6}$$

3. The Fraction of Amplification-biased Events

Due to amplification bias, each individual bright star works effectively as multiple source stars. Let $\mathcal{B}$ the effective number of source stars per single bright star: amplification bias factor. This factor is computed by

$$\mathcal{B}(L_b) = 1 + \int_0^{L_b} dL \Phi(L) \beta_{\max}(L_b, L) \langle \eta \rangle, \tag{3.1}$$

where $\Phi(L)$ is the LF of Galactic bulge stars normalized to the area of the seeing disk around each bright star. The average seeing of the current experiments is $\sim 2''$. However, when a faint source is located at the edge of the seeing disk of the bright star, one can isolate its magnified image from the bright star. I therefore set the undistinguishable separation between images at $\Delta\theta = 1''.5$, i.e., $\Phi(L)$ is normalized for stars in the area $\pi(1''.5)^2$. In equation (3.1) the value $\beta_{\max}$ is included because only events with $\beta < \beta_{\max}$ can be detected. In addition, the factor $\langle \eta \rangle = \int_0^{\beta_{\max}} \eta(\beta)d\beta/\beta_{\max}$ enters because the detection efficiency, ϵ, decreases as the time scale decreases. For the moment I assume that the efficiency is linearly proportional to the time scale. The adopted values of $\Delta\theta$ and the functional form $\epsilon(t_{\mathrm{e}})$ is subject to some uncertainties, so I will discuss other cases in § 4. Then the total effective number of source stars is obtained by integrating $\mathcal{B}(L_b)$ for all bright source stars weighted by the LF;

$$\mathcal{B}_{\mathrm{tot}} = \int_{L_{DL}}^\infty dL_b \Phi(L_b) \mathcal{B}(L_b)$$

$$= \int_{L_{DL}}^\infty dL_b \Phi(L_b) + \int_{L_{DL}}^\infty dL_b \Phi(L_b) \int_0^{L_b} dL \Phi(L) \beta_{\max}(L_b, L) \langle \eta \rangle. \tag{3.2}$$

The first and second terms in equation (3.2) are the number of bright stars in the seeing disk and the additional faint stars that effectively work as source stars due to the amplification bias effect, respectively.

The model LF is constructed as follows. For stars brighter than the de-reddened I-band mag of $I_0 = 18.2$, I adopt the LF determined by J. Frogel (1996, private communication), and in the range $18.2 \leq I_0 \leq 22.4$, I use the LF determined by Light et al. (1996). To extend the LF beyond even this limit, I adopt the LF of stars in the solar neighborhood determined by Gould, Bahcall, & Flynn (1996). I address below the uncertainty caused by the difference in the bulge and disk stellar populations. I adopt a distance $R_0 = 8.0$ kpc to the Galactic bulge stars. The model LF is presented in Figure 1 in units of stars mag^{-1} arcmin^{-2}. Current experiments reach the detection limit when

the stellar number density of Galactic bulge fields arrives at $\sim 10^6$ stars deg^{-2} (C. Alcock 1996, private communication). Based on the model LF this number density corresponds to $I_0 = 18.2$ mag or $M_I = 3.7$.

With the model LF and the corresponding detection limit, the amplification bias factor $\mathcal{B}$ is computed by equation (3.1) along with equations (2.2), (2.5), and (2.6). [1] The resultant values of $\mathcal{B}$ are shown as a function of bright star mag I_b in Figure 2. Because the LF increases as L_b decreases, combined with the fact that the most probable faint events have unamplified faint-star flux just below L_b, the value $\mathcal{B}$ increases as L_b decreases. Since the event rate is directly proportional to the number of source stars, the average increase in event rate due to the amplification bias effect is determined by

$$\langle \mathcal{B} \rangle = \frac{\mathcal{B}_{\mathrm{tot}}}{\int_{L_{DL}}^{\infty} dL_b \Phi(L_b)}, \tag{3.3}$$

resulting in $\langle \mathcal{B} \rangle = 1.65$. That is, on average each source star works effectively as ~ 1.7 stars, and thus the event rate increases by the same factor.

The distribution of faint source-star brightness for a given template star with a flux L_b is obtained by taking the derivative of $\mathcal{B}$, i.e.,

$$\frac{d\mathcal{B}(L_b, L)}{dL} = \delta(L - L_b) + \beta_{\max}(L_b, L)\langle \eta \rangle \Phi(L). \tag{3.4}$$

Note that $\int_{L_{DL}}^{\infty} dL \delta(L - L_b) = 1$. The distributions $d\mathcal{B}(L_b, L)$ for bright stars with $I_b = 15, 16, 17, 18$ mag are shown in the left panel of Figure 3. The total event distribution for all bright stars is computed by

$$d\mathcal{B}_{\mathrm{tot}}(L) = \int_{L_{DL}}^{\infty} dL_b \Phi(L_b) d\mathcal{B}(L_b, L)$$

$$= \Phi(L_b)dL_b + \int_{L_{DL}}^{\infty} dL_b \Phi(L_b) \left[\beta_{\max}(L_b, L)\langle \eta \rangle \Phi(L)dL \right], \tag{3.5}$$

where the first and second terms represent the event rate distribution without and with the amplification bias effect, respectively. When the amplification bias effect is not included,

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[1] An additional threshold for detection might be imposed by the flux from other stars located in the seeing disk, background flux. However, flux from stars much fainter than L_b will spread smoothly over the seeing disk and it will be subtacted during image process. Some stars just below L_b that are located in the seeing disk will cause the threshold to be higher. On the other hand, stars that are near but not inside the seeing disk will make the bright star appear fainter and so cause a lower threshold. To lowest order, these two effects cancel one another.

the distribution is just proportional to the LF, i.e., only the first term. The total event distribution is shown in the right panel of Figure 3, in which the contribution to the distribution by faint events are shaded. The faint events comprises $\sim 40\%$ of the total events and the unamplified mag of faint source stars extends up to $I_0 \sim 23$. For very faint stars, i.e., $I_0 \gtrsim 23$, to be detected they should be highly amplified, and thus they are rare. The slim chance of high amplification becomes even slimmer because of the drop in efficiency. Therefore, the uncertainty in the model LF in the very faint region does not affect the determination of $\langle \mathcal{B} \rangle$.

4.   Effect of the Amplification Bias on the Time Scale Distribution

Amplification bias has a major influence on the Einstein time scale distribution $f(t_{\mathrm{e}})$ because one measures t_{eff} instead of t_{e}. How and how much is the time scale distribution affected? To answer this question I construct time scale distributions for events toward the Galactic bulge with and without including the effect of amplification bias using a reasonable mass spectrum model of lenses. For the construction of $f(t_{\mathrm{e}})$, it is required to model the mass and velocity distributions. For the Galactic bulge mass distribution, I adopt a "revised COBE" model that is based on the triaxial COBE model (Dwek et al. 1995) except for the central part of the bulge. In the inner ~ 600 pc of the bulge, I adopt the centrally concentrated axisymmetric Kent model (1992) since the COBE model does not match well in this region. The model is provided in terms of the light density, ν. For the disk, I adopt a Gould et al. (1996) model which has of the form

$$\rho(R, z) = \rho_0 \left[\frac{4}{5} \operatorname{sech}^2 \left(\frac{z}{h_1} \right) + \frac{1}{5} \exp \left(-\frac{z}{h_2} \right) \right] \exp \left(-\frac{R - R_0}{3000} \right), \qquad (4.1)$$

where $h_1 = 323$ pc and $h_2 = 660$ pc, and $\rho_0 = 0.436 \ M_\odot \mathrm{pc}^{-3}$. Both the disk and bulge MACHO transverse speed is modeled by a Gaussian. In the model, the velocity distributions of disk MACHOs have means and standard deviations of $(\bar{v}_y, \sigma_y) = (220, 30)$ km s^{-1} and $(\bar{v}_z, \sigma_z) = (0, 20)$ km s^{-1}. The projected components of the Galactic bulge velocity dispersion are computed from the tensor virial theorem and results in $(\bar{v}_y, \sigma_y) = (0, 93.0)$ km s^{-1} and $(\bar{v}_z, \sigma_z) = (0, 78.6)$ km s^{-1}. Here the projected coordinates (y, z) are set so that the axes are respectively parallel and normal to the Galactic plane. For more details of the mass and velocity distributions, see Han & Gould (1995, 1996).

There may be dark lenses as well as lenses from known stellar populations. The mass spectrum, $f(M_L)$, of the stellar population is constructed by using the mass-luminosity relation provided by equation (5) of Henry & McCarthy (1993). The white dwarf component in the mass range $0.5 \ M_\odot \leq M_L \leq 0.7 \ M_\odot$ is included by normalizing its mass spectrum so that there are ~ 10 times more white dwarfs than the number of turnoff plus giant stars. Finally, brown dwarfs in the mass range $0.07 \ M_\odot \leq M_L \leq 0.09 \ M_\odot$ are included in the mass spectrum making up the rest of the total bulge mass of $M_{\mathrm{bulge}} = 2.1 \times 10^{10} M_\odot$, which is adopted from Zhao, Spergel, & Rich (1995). Then the total mass of each lens population i in the Galactic bulge is determined by

$$M_{\mathrm{pop},i} = M_{\mathrm{BW},i} \left(\frac{L_{\mathrm{bulge}}}{L_{\mathrm{BW}}} \right), \tag{4.2}$$

where $L_{\mathrm{bulge}} = \int_{\mathrm{bulge}} dxdydz \ \nu(x,y,z) = 1.8 \times 10^{10} \ L_\odot$ and $L_{\mathrm{BW}} = \int_{\ell_{\mathrm{BW}}} dD_{\mathrm{ol}} \nu(D_{\mathrm{ol}}) = 2412 \ L_\odot \ \mathrm{pc}^{-2}$ is the total amount of light in the bulge and the integrated light seen through a unit area (pc^2) of the Baade's Window (BW) (Kent 1992; Dwek et al. 1995). The integrated mass of each population is obtained from the mass spectrum model, i.e., $M_{\mathrm{BW},i} = \int dM_L f_i(M_L) M_L$. With the mass spectrum model I find that $M_{\mathrm{BW},i} = 1579, 339$, and $897 \ M_\odot \ \mathrm{pc}^{-2}$, and the resulting total masses in the bulge are $M_{\mathrm{pop},i} = 1.18 \times 10^{10}$, 0.25×10^{10}, and $0.67 \times 10^{10} \ M_\odot$ for the stellar, white dwarf, and brown dwarf populations, respectively.

With these models, the event rate distribution of bulge-bulge self-lensing and disk-bulge events for a *single* source star is computed by equations (3.3) and (3.4) of Han & Gould (1996). The lens masses are drawn from the model mass function. However, the COBE bulge model is given in terms of light density, ν, and thus the conversion from ν to n is required for proper normalization. This number-to-light density ratio is determined by

$$\frac{n}{\nu} = \frac{N_{\mathrm{BW}}}{L_{\mathrm{BW}}}. \tag{4.3}$$

Here $N_{\mathrm{BW}} = \sum_i N_{\mathrm{BW},i}$ is the total number of objects in the unit area of sky toward BW, in which the number of individual component is obtained by $N_{\mathrm{BW},i} = \int dM_L \ f_i(M_L) = 3441$, 566, and 11357 objects pc^{-2} for the stellar, white dwarf, and brown dwarf populations, respectively. Once again the total amount of light in the same area of sky is $L_{\mathrm{BW}} = 2412 \ L_\odot \ \mathrm{pc}^{-2}$ [see below eq. (4.2)]. I find $n/\nu = 6.31$. Note that although brown dwarfs comprise only 32% of the total mass, they account for 74% of the total number density.

Once $f(t_{\mathrm{e}})$ is obtained, the total event rate as a function of time scale not including the amplification bias effect, $F_{\mathrm{w/o}}(t_{\mathrm{e}})$, by monitoring *all* source stars is obtained by multiplying

the total number stars, N_*, and the total amount of observation time, T, into $f(t_e)$;

$$F_{w/o}(t_e) = f(t_e)N_*T; \qquad N_*T = \left(\frac{\pi}{2\tau}\right) \sum_j^{N_{event}} \frac{t_{e,j}}{\epsilon(t_{e,j})}, \qquad (4.4)$$

where N_{event} is the actually detected number of events. Note that N_* is the number of stars in the template, and thus only resolved stars. The MACHO group reported $N_{events} = 39$ events in the first year bulge season, resulting in an optical depth of $\tau = 2.4 \times 10^{-6}$ for all types of stars (Alcock et al. 1996b), which implies $N_*T = 1.6 \times 10^9$ days.

On the other hand, the construction of the total time scale distribution with amplification bias, $F_{with}(t_{eff})$, requires additional processing which is described below. For a single bright star, the factor η is distributed as

$$g(\eta) = \int_{L_{DL}}^{\infty} dL_b \Phi(L_b) \int_0^{L_b} dL \Phi(L) \int_0^{\beta_{max}} d\beta \ \delta\left[\eta - \left(\beta_{max}^2 - \beta^2\right)^{1/2}\right] \left[\int_{L_{DL}}^{\infty} dL_b \Phi(L_b)\right]^{-1}. \qquad (4.5)$$

Once $g(\eta)$ is obtained, the effective time scale distribution of faint events $F_f(t_{eff})$ is obtained from $F_{w/o}(t_e)$ by

$$F_f(t_{eff}) = \int_0^1 d\eta g(\eta) \int_0^{\infty} dt_{eff} F_{w/o}(t_e)\delta(t_{eff} - \eta t_e). \qquad (4.6)$$

Then, one finds the total (both bright and faint) event time scale distribution by $F_{with}(t_e) = F_{w/o}(t_e) + F_f(t_e)$. With the detection efficiency $\epsilon(t_e)$ provided by Alcock et al. (1996b), the final time scale distributions with and without amplification bias are computed by $\Gamma_{w/o}(t_e) = \epsilon(t_e)F_{w/o}(t_e)$ and $\Gamma_{with}(t_e) = \epsilon(t_e)F_{with}(t_e)$, and they are shown in Figure 4. In the figure, the distributions are compared with the observed time scale distribution (shaded histogram) obtained by the MACHO group (Alcock et al. 1996b).

There are two major changes in the time scale distribution by taking the amplification bias effect into account. First, the distribution $\Gamma_{with}(t_e)$, in general, has a higher normalization relative to $\Gamma_{w/o}(t_e)$ due to the increase in number of stars that are effectively monitored. The increase factor is $\langle \mathcal{B} \rangle = \int dt_e \Gamma_{with}(t_e)/ \int dt_e \Gamma_{w/o}(t_e) = 1.65$, which matches well with the value determined in § 3 using the approximation $\epsilon(t_e) \propto t_e$. Second, $\Gamma_{with}(t_e)$ is shifted toward shorter time scale compared to $\Gamma_{w/o}(t_e)$ due to the additional contribution by faint events which mainly have short time scales. Therefore, a significant fraction of short events (~ 10 days), which could not be explained by known lens populations, might be caused by the amplification bias effect.

However, there are some uncertainties in determining the contribution to the event rate by faint events. One of the uncertainties comes from the maximum size of separation

at which one can isolate the image of the amplified faint star from that of a bright star. I compute the increase factors in event rate for $\Delta\theta = 1''.0$ and $2''.0$ and find that $\langle \mathcal{B} \rangle = 1.35$, and 2.38, respectively. Additional uncertainty comes from the blending of bright stars with faint stars. Due to the blending, some fraction of bright events would fail to be detected and the measured time scale would be shorter, resulting lower normalization and additional shift toward shorter time scale in $\Gamma(t_{\mathrm{e}})$. However, because of the dominance of the bright star flux, this effect would also be small.

5.   The Effects of Amplification Bias on Optical Depth Determination

When the amplification bias is not taken into consideration the optical depth might be overestimated by a factor $\langle \mathcal{B} \rangle \sim 1.7$ because events are detected by monitoring $N_{*,\mathrm{eff}} = \langle \mathcal{B} \rangle N_*$, while τ is determined with N_* instead of $N_{*,\mathrm{eff}}$. On the other hand, the effects of amplification bias on t_{e} and ϵ do not propagate to the optical depth determination. This is because by measuring t_{eff} instead of t_{e}, the time scale decreases by a factor $\sim \eta$ and for the same reason the detection efficiency decreases by a similar factor, i.e., the same factors cancel each other out and thus there is no net effect. In addition, most additional events due to amplification bias are expected to have short t_{eff} in which the efficiency is well approximated as linear.

6.   Conclusion

I have shown that amplification bias may have significant effects on the time scale distribution and the optical depth determination, and thus the correction of the bias is very important. The true distribution $\Gamma_{\mathrm{w/o}}(t_{\mathrm{e}})$ can be recovered statistically with the known weight factor $d\mathcal{B}_{\mathrm{tot}}(L)$ in the reverse way that Γ_{with} is obtained from $\Gamma_{\mathrm{w/o}}$. In addition, for some long t_{eff} events for which very detailed light curves can be constructed, it will be possible to recover the individual true t_{e} by the fitting the light curves using an additional parameter, the unmagnified flux. Another way to detect biased events is finding the shift of the centroid of source stars caused by the faint star amplification. This shift was actually detected by Alard (1996) despite the moderate quality of the image: photographic plate. An inspiring development in the lensing experiments is that the time

resolution of observations is improving rapidly. For example, the alert system allows intensive network observations of candidate events (PLANET, Albrow et al. 1996; GMAN, Pratt 1996). The EROS group (Aubourg et al. 1995) has carried out a lensing experiment toward the Magellanic Clouds with a monitoring frequency of up to 46 times per night. In this way, it will be possible to obtain detailed light curves, and so t_e for a significant fraction of individual events.

REFERENCES

Alard, C. 1996, in Proc. IAU Symp. 173, Astrophysical Applications of Gravitational Lensing, ed. C. S. Kochanek & J. N. Hewitt, (Dordrecht: Kluwer), 215

Albrow, M. et al. 1996, in Proc. IAU Symp. 173, Astrophysical Applications of Gravitational Lensing, ed. C. S. Kochanek & J. N. Hewitt, (Dordrecht: Kluwer), 237

Alcock, C., et al. 1993, Nature, 365, 621

Alcock, C., et al. 1996a, ApJ, 463, L67

Alcock, C., et al. 1996b, ApJ, submitted

Alcock, C., et al. 1996c, ApJ, submitted

Aubourg, E., et al. 1993, Msngr, 72, 20

Aubourg, E., et al. 1995, A&A, 301, 1

Bennett, D. P., et al. 1994, in AIP Conf. Proc. 316, Dark Matter, ed. S. Holt & C. Bennett (New York: AIP), 77

Blanford, R., & Narayan, R. 1992, ARA&A, 30, 311

Bouquet, A. 1993, A&A, 280, 1

Dwek, E., et al. 1995, ApJ, 445, 716

Gould, A., Bahcall, J. A., & Flynn, C. 1996, ApJ, 465, 759

Han, C., & Gould, A. 1995, 447, 53

Han, C., & Gould, A. 1996, 467, 000

Henry, T. J., & McCarthy, D. W. 1993, AJ, 106, 773

Kent, S. M. 1992, ApJ, 387, 181

Light, R. M., Baum, W. A., & Holtzman, J. A. 1996, in preparation

Narayan, R., & Wallington, S. 1994, in Proc. 31st International Liege Colloquium: Gravitational Lensing in the Universe (Liege: Inst. d'Astrophysique), 217

Nemiroff, R. J. 1994, ApJ, 435, 682

Mao, S., & Paczyński, B. 1995, AcA, 45, 349

Pratt, M. R., et al. 1996, in IAU Symp. 173, Astrophysical Applications of Gravitational Lensing, ed. C. S. Kochanek & J. N. Hewitt, (Dordrecht: Kluwer)

Udalski, A., et al. 1992, AcA, 42, 253

Udalski, A., et al. 1994, AcA, 44, 165

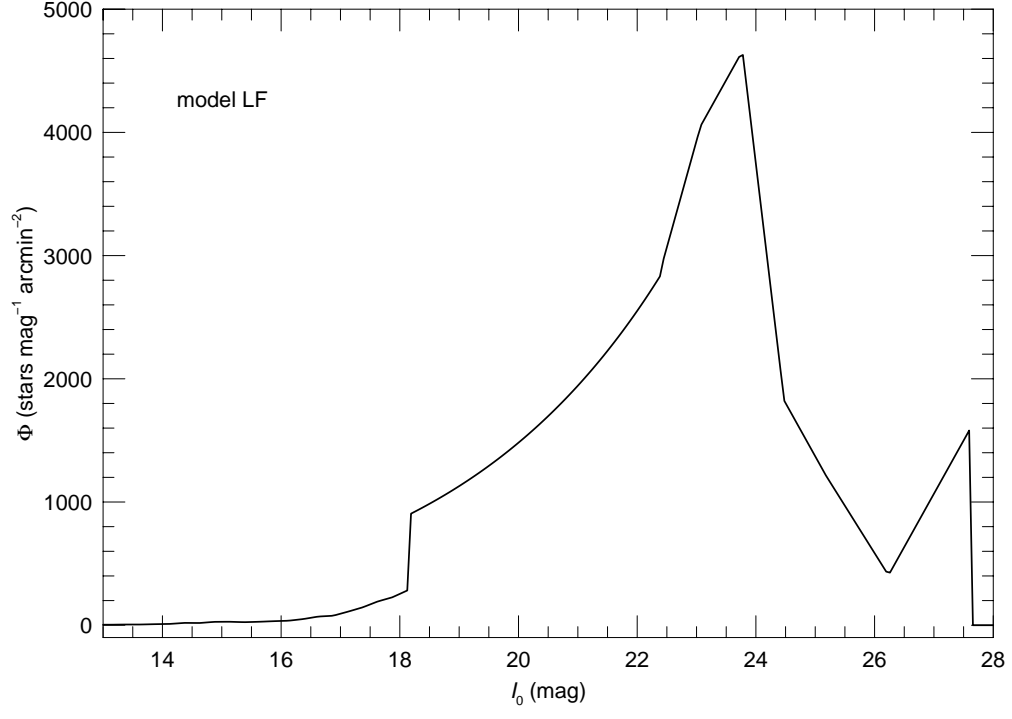Zhao, H. S., Spergel, D. N., & Rich, R. M. 1995, ApJ, 440, L13

Figure 1: The model luminosity function in de-reddened *I*-band.
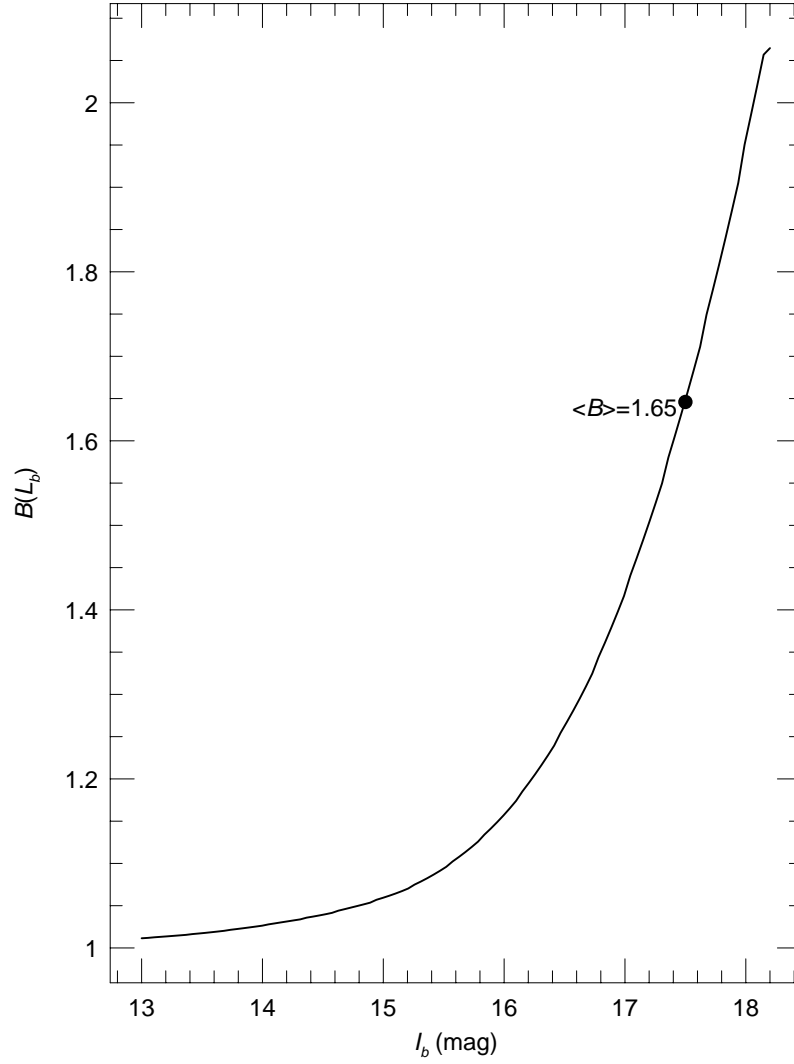
Figure 2: The amplification bias factors as a function of bright source star mag. Bacause the LF of bright stars increases, combined with the fact that the most probable faint events have unamplified faint-star flux just below L_b, the value $\mathcal{B}$ increases as L_b decreases.
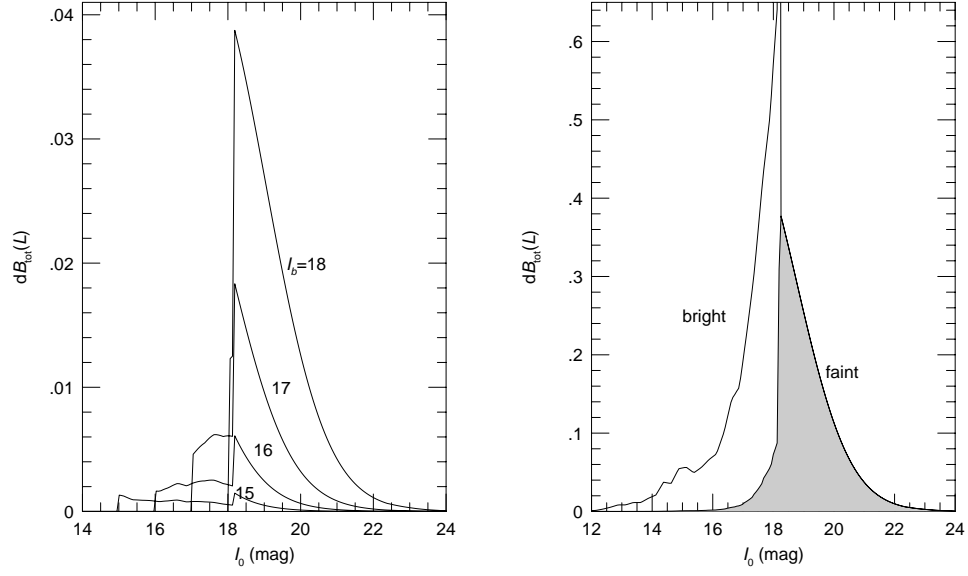
Figure 3: The distributions of faint source-star brightness for bright star mag of $I_b = 15$, 16, 17, and 18. Also shown are the total (both bright and faint) event distributions as functions of source star mag. The contribution by faint events is shaded. Under the approximation that $\epsilon \propto t_e$, faint events comprise 40% of total events.
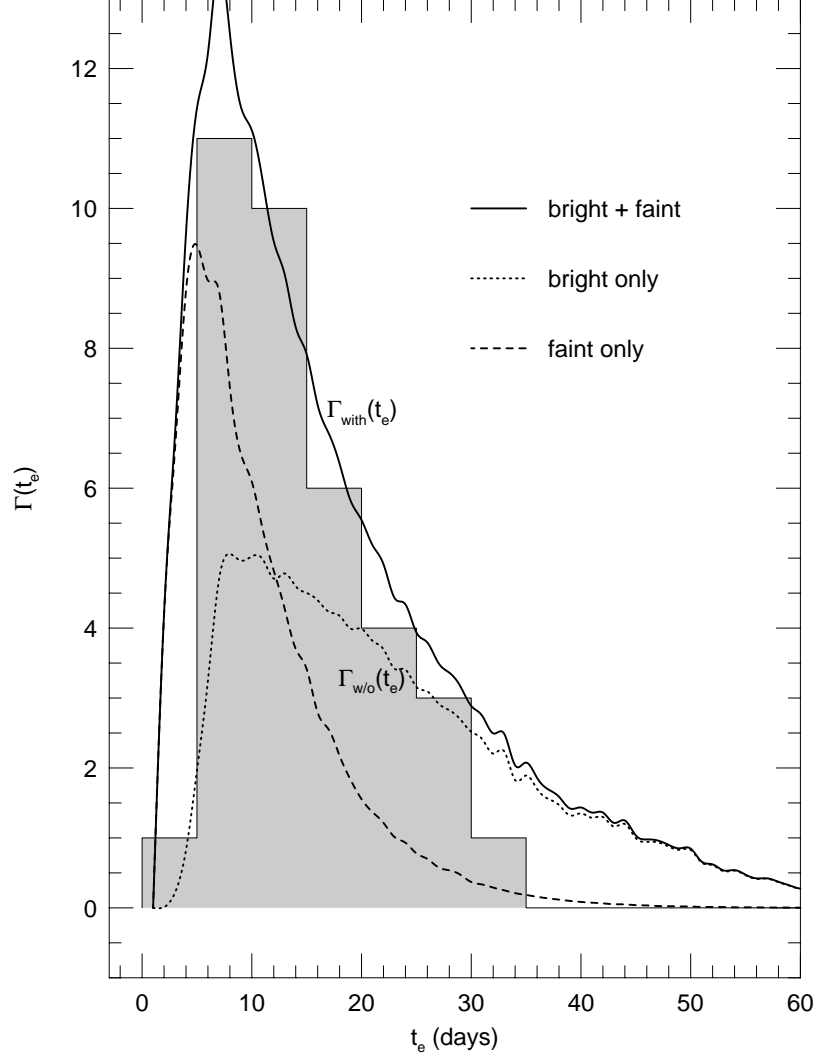
Figure 4: The time scale distributions for *all* stars with and without including amplification bias effect, and they are compared with the observed distribution obtained by the MACHO group (shaded histogram).